## Addition vocabulary:

## To add successfully, children need to be able to:

- recall all addition pairs to 9+9 and complements in 10
- add mentally a series of one-digit numbers, such as $5+8+4$
- add multiples of 10 (such as $60+70$ ) or of 100 (such as $600+700$ ) using the related addition fact, $6+7$, and their knowledge of place value
- partition two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways


## Written methods for addition of whole numbers

## Stage 1: The empty number line

## Method

The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. The empty number line helps to record the steps on the way to calculating the total.

## Example

Steps can be recorded on a number line. The steps often bridge through a multiple of 10 .

$$
34+23=57
$$



## Stage 2: Partitioning

## Method

The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.
Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

## Example

Record steps in addition using partitioning:
$47+76=47+70+6=117+6=123$
$47+76=40+70+7+6=110+13=123$
Partitioned numbers are then written 70
one under another where the most 40
significant number is written first: 7
$+6$

## Stage 3: Expanded method in columns

## Method

Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums either the tens or the ones can be found in any order. As children gain confidence, ask them to start by adding the ones digits first always.
The addition of the tens in the calculations $47+76$ is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.
The expanded method leads children to the more compact method so that they understand its structure and efficiency.

## Example

Write the numbers in columns.
Adding the tens first:

Discuss how adding the ones first give the same answer as adding the tens first. Refine over time to adding the ones first consistently.

## Stage 4: Column method

## Method

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

Later, extend to adding three two digit numbers and numbers with different numbers of digits.

## Example

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, method is quick and reliable.

| 587 |  |
| ---: | ---: |
| +475 |  |
| 1062 | 3587 |
| 11 | $+\quad 675$ |
| 4262 |  |
| 111 |  |

## Subtraction vocabulary: Difference between, subtract, fewer than, decrease, take away, minus, reduce

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20
- subtract multiples of 10 (such as $160-70$ ) using the related subtraction fact, 16-7, and their knowledge of place value
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into $70+4$ or $60+$ 14)


## Written methods for subtraction

## Stage 1: Using the empty number line

## Method

The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74-27 can be recorded by counting back 27 from 74 to reach 47.The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 12, 86-77 or 43-28.

## Example

Steps can be recorded on a number line. The steps often bridge through a multiple of 10.

Counting back is used:
The steps may recorded in a different order or combined:

$$
47-23=24
$$



Stage 2: Partitioning

## Method

Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 89-57 this involves partitioning the 57 into 50 and 7, then subtracting from 89 the 50 and the 7 in turn.
This requires children to subtract a single-digit number or a multiple of 10 from a two digit number mentally. The method of recording links to counting back on the number line. Partitioning the numbers into tens and units and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.

## Example

$89-57=89-50-7=32$

| 89 |
| ---: |
| -57 |
| -30 |$\quad$| 80 |
| :--- |
| 50 |$=32$

## Stage 3: Expanded layout leading to column method

## Method

Partitioning is combined with decomposition in the expanded layout to represent calculations which use greater numbers where the units and tens to be taken away are greater than the units and tens in the original amount. Children adjust from T to U and from H to T . Use of concrete resources such as place value counters should be used in the first instance to represent the 'exchanging' process.

Example


## Stage 4: Column method

Method
Children use the decomposition method to condense their calculation
only once they understand the expanded method and the value of each
digit in the calculation.
Where the numbers are involved in the calculation are close together
or near to multiples of 10, 10 etc. mental methods such as counting on
using a number line should be used.

Click here to watch a short video of children demonstrating their methods: Subtraction

Multiplication vocabulary: multiplied by, multiply, product, groups of, lots of, times table, times $x$

To multiply successfully, children need to be able to:

- recall all multiplication facts to $12 \times 12$
- partition number into multiples of one hundred, ten and one
- work out products such as $70 \times 5,70 \times 50,700 \times 5$ or $700 \times 50$ using the related fact $7 \times 5$ and their knowledge of place value
- add two or more single-digit numbers mentally
- add multiples of 10 (such as $60+70$ ) or of 100 (such as $600+$ 700) using the related addition fact, $6+7$, and their knowledge of place value
- add combinations of whole numbers using the column method
- multiply and divide whole numbers and decimals by 10/100/1000

Stage 1: Repeated addition and arrays

## Method

Bead strings, objects etc. and jottings (on a number line) are used to support early understanding of multiplication as repeated addition.
Children should recognise that in a multiplication calculation the position of each number is interchangeable (commutivity). Arrays are used to model a multiplication calculation which feeds into more complex strategies.

## Example

Stage 2: Mental multiplication using partitioning

## Method

Mental methods for multiplying TU $\times U$ are a useful starting point. This should be linked to repeated addition. Tens and units are multiplied separately and then recombined.
It is more common to deal with the largest number first.

## Example

```
38\times5=(30\times5)+(8\times5)
```

    \(=150+40\)
    \(=190\)
    Stage 3: The grid method

## Method

## Example

The method is a visual representation of stage 2 . The number with most digits is placed in the left hand column. Each number is partitioned, separated calculations take place and the total is found. The method requires children to confidently multiply by 10 , find the total of more than 2 numbers, partition into tens and units.

This method can be developed to solve TU x TU, HTU x U and beyond to U.t $x U$ and TU.t $x U$

$5 \times 3=5+5+5$
00000 $00000 \quad 5 \times 3=15$ 00000 $3 \times 5=15$

$5 \times 3=5+5+5$



To divide successfully, children need to be able to:

- understand and use the vocabulary of division - for example in $18 \div 3=$ 6 , the 18 is the dividend, the 3 is the divisor and the 6 is the quotient
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways
- recall multiplication and division facts to $12 \times 12$, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
- know how to find a remainder working mentally - for
example, find the remainder when 48 is divided by 5
- understand and use multiplication and division as inverse operations.
- understand division as repeated subtraction
- estimate how many times one number divides into another -
for example, how many sixes there are in 47 , or how many 23 s there are in 92
- multiply a two-digit number by a single-digit number
- subtract numbers mentally and using the column method
- multiply and divide whole numbers and decimals by 10/100/1000


## Written methods for division

## Stage 1: Repeated subtraction using a number line

## Method

Children should be able to find a remainder mentally, for example the remainder when 34 is divided by 6 using number lines to record jottings.

Children need to be able to count up and backwards confidently in different intervals.

Knowledge of times tables and division tables is essential.

Use of a number line to show repeated subtraction

Children can then move on to subtracting multiples of the divisor

Including calculations where there is a quantity 'left over'

## Example



## Stage 2: TU $\div \mathbf{U}$

## Method

Short division - 'chunking' - can be introduced to children who are confident in their knowledge of times tables and division facts up to at least $10 \times 10$.

Children should be able to subtract multiples of ten mentally, and have a sound understanding of partitioning and place value.


## Example



## Stage 3: HTU $\div$ TU

## Method

'Chunking' method is used to calculate HTU $\div$ TU.
Knowledge of timestables and division tables is crucial to have a secure understanding of the value of the numbers being worked with. Children should be able to multiply by multiples of 10 and subtract using the compact method.
This is extended to TU or HTU $\div$ U.t where children are secure in being able to interpret the remainder as a fraction of the whole amount. Understanding of fractions is key at this stage.

## Stage 4: Short division - compact

## Method

Compact division is used for HTU $\div \mathrm{U}, \mathrm{TU} \div \mathrm{U}$ (including where the answer will have a remainder) and moving on questions and answers involving decimals, when children's understanding of place value is secure and each earlier stage can be completed confidently. It is not an alternative method but a progression. Children should make sensible decisions about the interpretation of the remainder and be able to put such calculations into context.

## Example



## Example



